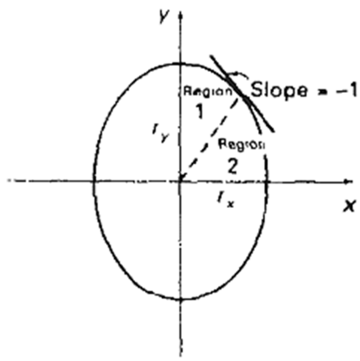
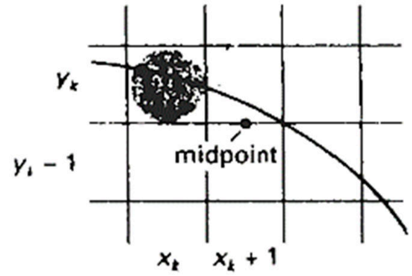
MIDPOINT ELLIPSE





$$\left(\frac{x - x_c}{r_x}\right)^2 + \left(\frac{y - y_c}{r_y}\right)^2 = 1$$

. we take the start position at $(0, r_y)$ and step along the ellipse path in clockwise order throughout the first quadrant.

We define an ellipse function from Eq. 3-35 with $(x_c, y_c) = (0, 0)$ as

$$f_{\text{ellipse}}(x, y) = r_y^2x^2 + r_x^2y^2 - r_x^2r_y^2 \tag{3-37}$$

which has the following properties:

$$f_{\text{ellipse}}(x, y) \begin{cases} < 0, & \text{if } (x, y) \text{ is inside the ellipse boundary} \\ = 0, & \text{if } (x, y) \text{ is on the ellipse boundary} \\ > 0, & \text{if } (x, y) \text{ is outside the ellipse boundary} \end{cases} \tag{3-38}$$

Starting at $(0, r_y)$, we take unit steps in the x direction until we reach the boundary between region 1 and region 2 (Fig. 3-20). Then we switch to unit steps in the y direction over the remainder of the curve in the first quadrant. At each step, we need to test the value of the slope of the curve. The ellipse slope is calculated from Eq. 3-37 as

$$\frac{dy}{dx} = -\frac{2r_y^2x}{2r_x^2y} \tag{3-39}$$

At the boundary between region 1 and region 2, $dy/dx = -1$ and

$$2r_y^2x = 2r_x^2y$$

Therefore, we move out of region 1 whenever

$$2r_y^2x \geq 2r_x^2y \tag{3-40}$$

Assuming position (x_k, y_k) has been selected at the previous step, we determine the next position along the ellipse path by evaluating the decision parameter (that is, the ellipse function 3-37) at this midpoint:

$$p1_k = f_{\text{ellipse}}\left(x_k + 1, y_k - \frac{1}{2}\right)$$
$$= r_y^2(x_k + 1)^2 + r_x^2\left(y_k - \frac{1}{2}\right)^2 - r_x^2 r_y^2 \qquad (3\text{-}41)$$

If $p1_k < 0$, the midpoint is inside the ellipse and the pixel on scan line y_k is closer to the ellipse boundary. Otherwise, the midposition is outside or on the ellipse boundary, and we select the pixel on scan line $y_k - 1$.

REGION1

$$p1_{k+1} = f_{\text{ellipse}}\left(x_{k+1} + 1, y_{k+1} - \frac{1}{2}\right)$$
$$= r_y^2[(x_k + 1) + 1]^2 + r_x^2\left(y_{k+1} - \frac{1}{2}\right)^2 - r_x^2 r_y^2$$

or

$$p1_{k+1} = p1_k + 2r_y^2(x_k + 1) + r_y^2 + r_x^2\left[\left(y_{k+1} - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2\right] \qquad (3\text{-}42)$$

where y_{k+1} is either y_k or $y_k - 1$, depending on the sign of $p1_k$.

$$\text{increment} = \begin{cases} 2r_y^2 x_{k+1} + r_y^2, & \text{if } p1_k < 0 \\ 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 y_{k+1}, & \text{if } p1_k \geq 0 \end{cases}$$

In region 1, the initial value of the decision parameter is obtained by evaluating the ellipse function at the start position $(x_0, y_0) = (0, r_y)$:

$$p1_0 = f_{\text{ellipse}}\left(1, r_y - \frac{1}{2}\right)$$
$$= r_y^2 + r_x^2\left(r_y - \frac{1}{2}\right)^2 - r_x^2 r_y^2$$

or

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2 \qquad (3\text{-}45)$$

REGION 2

Over region 2, we sample at unit steps in the negative y direction, and the midpoint is now taken between horizontal pixels at each step (Fig. 3-22). For this region, the decision parameter is evaluated as

$$p2_k = f_{\text{ellipse}}\left(x_k + \frac{1}{2},\ y_k - 1\right) = r_y^2\left(x_k + \frac{1}{2}\right)^2 + r_x^2(y_k - 1)^2 - r_x^2 r_y^2 \qquad (3\text{-}46)$$

If $p2_k > 0$, the midposition is outside the ellipse boundary, and we select the pixel at x_k. If $p2_k \le 0$, the midpoint is inside or on the ellipse boundary, and we select pixel position x_{k+1}.

To determine the relationship between successive decision parameters in region 2, we evaluate the ellipse function at the next sampling step $y_{k+1} - 1 = y_k - 2$:

$$p2_{k+1} = f_{\text{ellipse}}\left(x_{k+1} + \frac{1}{2}, y_{k+1} - 1\right) = r_y^2\left(x_{k+1} + \frac{1}{2}\right)^2 + r_x^2[(y_k - 1) - 1]^2 - r_x^2 r_y^2 \qquad (3\text{-}47)$$

or

$$p2_{k+1} = p2_k - 2r_x^2(y_k - 1) + r_x^2 + r_y^2\left[\left(x_{k+1} + \frac{1}{2}\right)^2 - \left(x_k + \frac{1}{2}\right)^2\right] \qquad (3\text{-}48)$$

with x_{k+1} set either to x_k or to $x_k + 1$, depending on the sign of $p2_k$.

When we enter region 2, the initial position (x_0, y_0) is taken as the last position selected in region 1 and the initial decision parameter in region 2 is then

$$p2_0 = f_{\text{ellipse}}\left(x_0 + \frac{1}{2}, y_0 - 1\right) = r_y^2\left(x_0 + \frac{1}{2}\right)^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2 \qquad (3\text{-}49)$$

Midpoint Ellipse Algorithm

1. Input r_x, r_y, and ellipse center (x_c, y_c), and obtain the first point on an ellipse centered on the origin as

$$(x_0, y_0) = (0, r_y)$$

2. Calculate the initial value of the decision parameter in region 1 as

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$

3. At each x_k position in region 1, starting at $k = 0$, perform the following test: If $p1_k < 0$, the next point along the ellipse centered on (0, 0) is (x_{k+1}, y_k) and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$$

with

$$2r_y^2 x_{k+1} = 2r_y^2 x_k + 2r_y^2, \qquad 2r_x^2 y_{k+1} = 2r_x^2 y_k - 2r_x^2$$

and continue until $2r_y^2 x \geq 2r_x^2 y$.

4. Calculate the initial value of the decision parameter in region 2 using the last point (x_0, y_0) calculated in region 1 as

$$p2_0 = r_y^2 \left(x_0 + \frac{1}{2}\right)^2 + r_x^2 (y_0 - 1)^2 - r_x^2 r_y^2$$

5. At each y_k position in region 2, starting at $k = 0$, perform the following test: If $p2_k > 0$, the next point along the ellipse centered on (0, 0) is $(x_k, y_k - 1)$ and

$$p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p2_{k+1} = p2_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

using the same incremental calculations for x and y as in region 1.

6. Determine symmetry points in the other three quadrants.
7. Move each calculated pixel position (x, y) onto the elliptical path centered on (x_c, y_c) and plot the coordinate values:

$$x = x + x_c, \qquad y = y + y_c$$

8. Repeat the steps for region 1 until $2r_y^2 x \geq 2r_x^2 y$.